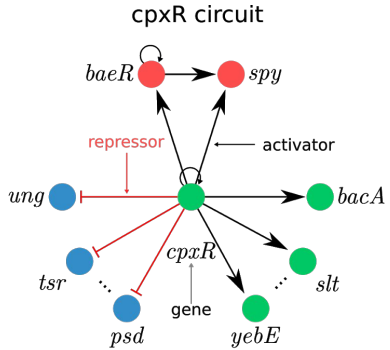
Fibration symmetries uncover the building blocks of biological networks

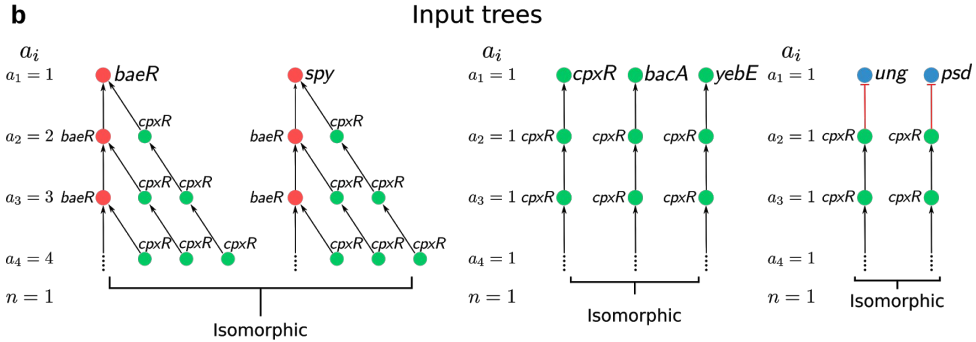
Ian Leifer, Flaviano Morone, Hernan Makse

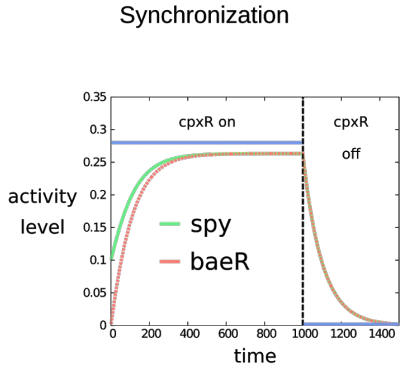
Introduction

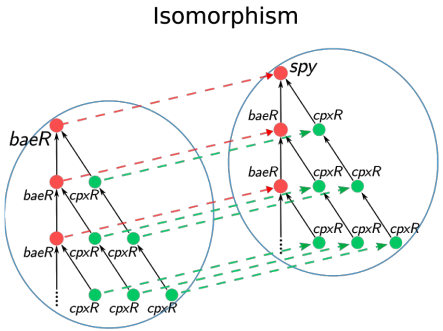
- Decomposing systems into building blocks provides a great way to study them
- Symmetry considerations provide a way to find building blocks in real large-scale networks
- The way building blocks function and interact can shed some light on how the complex behavior in networks emerges
- Here we talk about applications to transcriptional regulatory networks and use examples from bacteria, but this can be applied to any directed network

Methods. Input-trees, fibers, branching ratios, synchronization

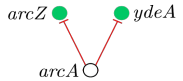
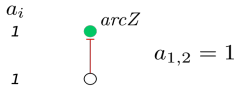
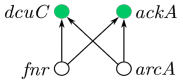
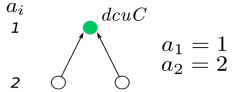
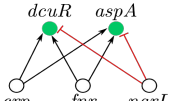
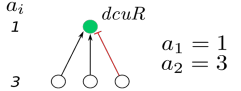


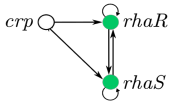
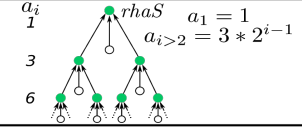
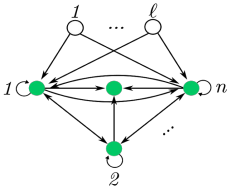
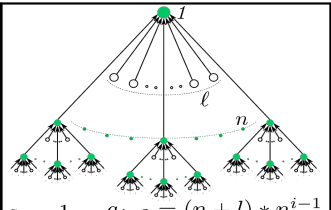
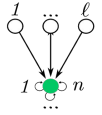




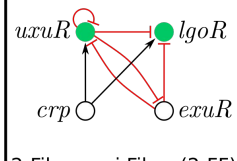
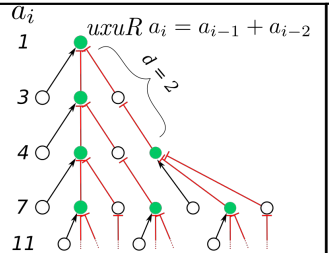
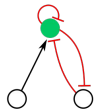
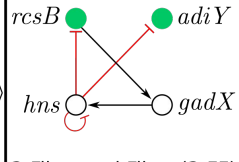
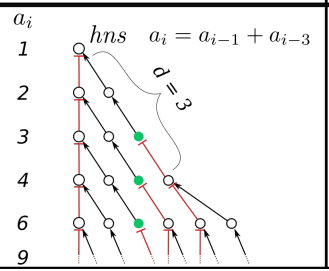
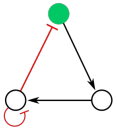


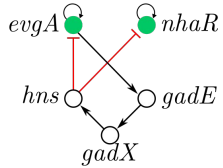
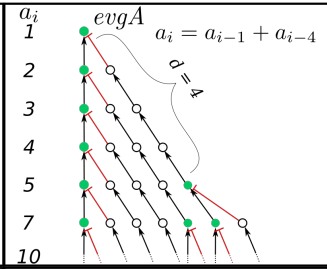
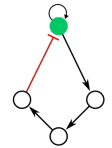
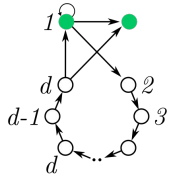
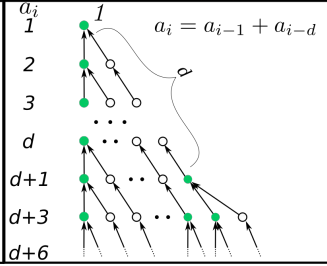
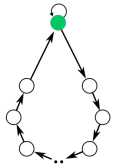
Building blocks. Integer branching ratios

$\|n, \ell\rangle$	Genetic circuit	Input tree	Base
$\|0, 1\rangle$			
$\|0, 2\rangle$			
$\|0, 3\rangle$			

$\|n, \ell\rangle$	Genetic circuit	Input tree	Base
$\|2, 1\rangle$			
$\|n \geq 3, \ell\rangle$			

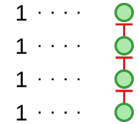
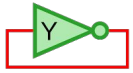
Building blocks. Fibonacci branching ratios

$\lvert\varphi_d, \ell\rangle$	Genetic circuit	Input tree	Base
$\lvert 1.6180..., 2\rangle$	 2-Fibonacci Fiber (2-FF)		
$\lvert 1.4655..., 1\rangle$	 3-Fibonacci Fiber (3-FF)		

$\lvert\varphi_d, \ell\rangle$	Genetic circuit	Input tree	Base
$\lvert 1.3802..., 1\rangle$	 4-Fibonacci Fiber (4-FF)		
$\lvert\varphi_d, \ell\rangle$	 d-Fibonacci Fiber (d-FF)		

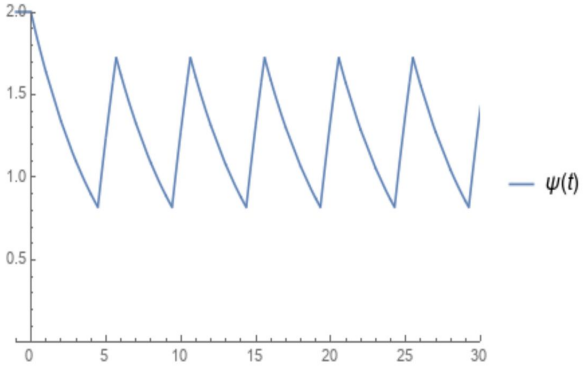
Analogy to electronic circuits (clock)

Repressor Autoregulator Loop $|1,0\rangle$

Genetic network	Q_t Input-tree	*Base*	Logic circuit
			

Continuous model

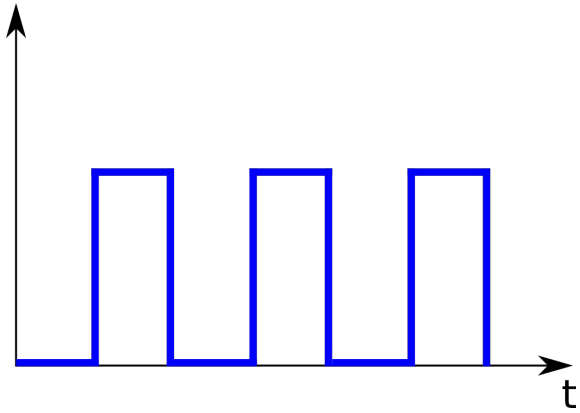
$$\dot{\psi} = -\alpha\psi(t) + \delta\ \theta(1 - \psi(t - \tau))$$



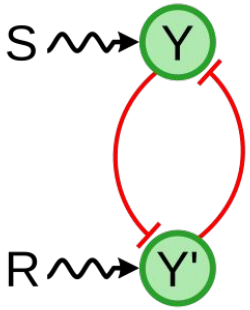
Boolean model

t	Y_t
0	1
1	0
2	1
3	0
4	1

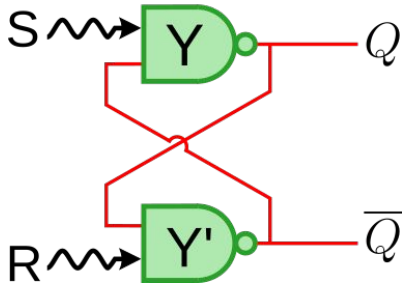
$Y_t = !Y_{t-1}$

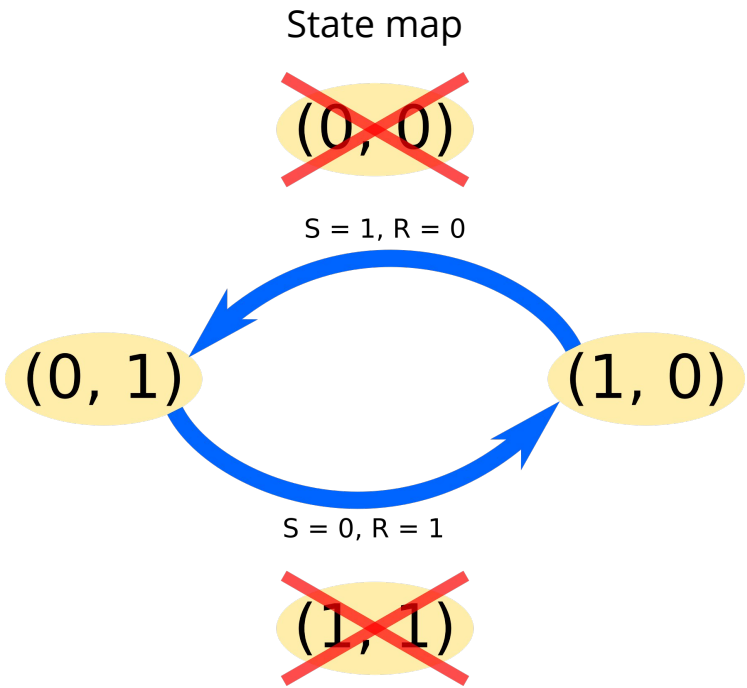


Analogy to electronic circuits (memory)



SR flip-flop





Conclusions

- Fibration symmetry is a type of symmetry that can provide the way to analyze biological (or any other directed) network
- Symmetries of the network help uncover new functional building blocks related to synchronization
- This provides a theoretically principled and algorithmically supported strategy to search for computational building blocks in biological networks

Further reading:
Morone, Leifer, Makse, PNAS (2020)
Leifer, et al. Plos. Comp. bio (2020)

Algorithm availability:
https://github.com/ianleifer/fibrationSymmetries
https://github.com/makselab